4. A. L. Kuznetsov, Tr. Leningr. Kopablestroite'. Inst., No. 10 (1964).
5. V. A. Bubnov, G. N. Den, and A. N. Shershneva, Tr. Tsentr. Nauchno-Issled. Proekt. -Konstrutor. Kotloturb. Inst. im. I. I. Polzunova, No. 89 (1968).
6. S. S. Evgen'ev, Author's Abstract of Candidate's Dissertation, Kazan' (1971).
7. B. J. Bayley and J. M. Oweh, "Flow between a rotating and a stationary disk," Aeronaut. Quart., 20, No. 4 (1969).
8. H. Schlichting, Boundary Layer Theory, McGraw-Hill, New York (1967).

BEHAVIOR OF FILM OF VISCOPLASTIC LIQUID IN THE PRESENCE OF SLIP AT THE WALL

Z. P. Shul'man, V. I. Baikov, and S. L. Benderskaya

UDC 532.501.32: 532.135

The shaking loose of a viscoplastic film of limiting thickness from a plane surface is considered for the case when there is effective slip at the wall.

A film of viscoplastic liquid is characterized by a limiting value of the thickness, at which no flow is observed under the action of gravity. This limiting thickness is found from the balance of frictional and gravitational forces

$$h=\tau_0/\rho g,$$

where τ_0 is the yield point; ρ is the density; and g is the acceleration of gravity.

In a number of technological processes, it is necessary to prevent the formation of a liquid film at a wall. The present paper considers a dynamic approach to this problem, by vibration of the wall.

Suppose that the wall and the adhering film are moving uniformly downward with velocity U and, at the initial moment t = 0, stop instantaneously. Close to the wall, the stress exceeds the yield point τ_0, which leads to the formation of a region of viscoplastic flow. In the second region, where the stress is less than τ_0, the liquid moves in a quasisolid manner. In the immediate vicinity of the wall, the moving disperse system, or polymer solution, may be separated into a thin layer of solvent, with respect to which all the remaining mass slips, as in a lubricant. It is possible to neglect the thickness of the region at the wall in comparison with the film thickness and to assume that at the surface of the plate the adhesion hypothesis does not hold, i. e., there is effective slip at the wall, $u(0, t) \neq 0$. (The case $u(0, t) = 0$ was considered in [1].)

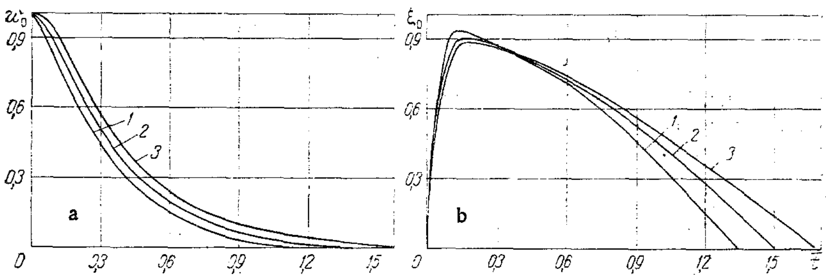

Fig. 1. Velocity of quasisolid core of film flow (a) and boundary of quasisolid region (b) for S = 0.25: 1) P = 0; 2) 0.05; 3) 0.1.

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk.
Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 32, No. 2, pp. 242-246, February, 1977. Original article submitted January 13, 1976.

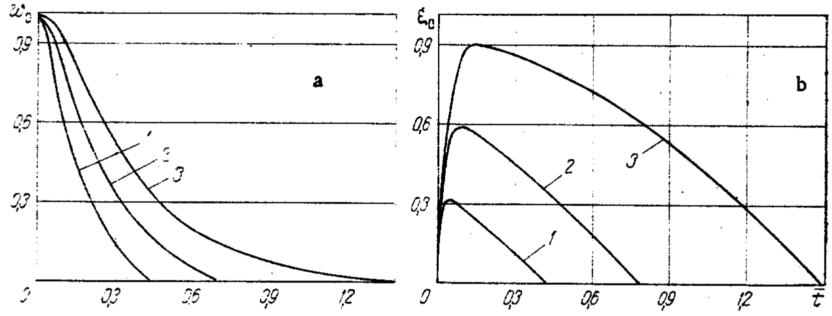

Fig. 2. Velocity of quasisolid core of film flow (a) and boundary of quasisolid region (b) for P = 0.05; 1) S = 10; 2) 2; 3) 0.25.

For a Shvedov–Bingham medium

$$\tau = \tau_0 + \mu_p \frac{du}{dy}$$

the equation of motion takes the form

$$\frac{\partial u}{\partial t} = \frac{\mu_p}{\rho} \frac{\partial^2 u}{\partial y^2} + g, \quad 0 \leqslant y \leqslant y_0(t), \tag{1}$$

$$\frac{\partial u}{\partial y} = 0, \quad y_0(t) \leqslant y \leqslant h, \tag{2}$$

where $y_0(t)$ is the moving boundary of the viscoplastic and quasisolid regions, the position of which must be determined in the course of the solution.

Integrating Eq. (2) gives

$$u = u_0(t), \quad y_0(t) \leqslant y \leqslant h.$$

The velocity of motion of the quasisolid region is found from the equation

$$\frac{du_0}{dt} = g - \frac{\tau_0}{\rho(h - y_0)}, \tag{3}$$

which is obtained from the balance of forces acting on an element of volume of the quasisolid core.

To determine the boundary conditions, assume that the rate of effective slip at the surface of the plate is of the form

$$\frac{1}{u(0,\ t)} = \frac{1}{U} + \frac{1}{k\dfrac{\partial}{dy}u(0,\ t)}$$

or

$$u(0,\ t) = \frac{kU\dfrac{\partial}{\partial y}u(0,\ t)}{U + k\dfrac{\partial}{\partial y}u(0,\ t)}, \tag{4}$$

where k is the slip coefficient.

In physical terms, Eq. (4) corresponds to the requirement that the effective rate of slip at the wall is finite at the initial moment [when $t \to 0$, $(\partial/\partial y)u(0, t) \to \infty$, $u(0, t) \to U$] and zero when $(\partial/\partial y)u(0, t) = 0$.

The conditions of continuity of stress and velocity at the moving boundary of the viscostatic and quasisolid regions $y = y_0(t)$ give

$$u[y_0(t),\ t] = u_0(t), \ \frac{\partial}{\partial y} u[y_0(t),\ t] = 0, \quad t > 0, \tag{5}$$

and the initial condition gives

$$u(y,\ 0) = U, \quad u_0(0) = U, \quad y_0(0) = 0, \quad 0 \leqslant y \leqslant h. \tag{6}$$

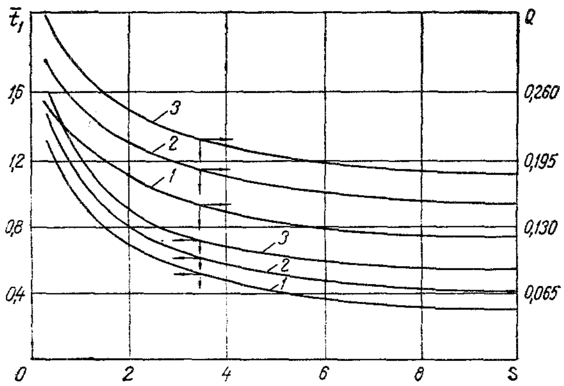

Fig. 3. Dependence of time of process $\bar{t}_1$ and mass of liquid shaken loose Q on S: 1) P = 0; 2) 0.05; 3) 0.1.

In the dimensionless variables

$$\xi = \frac{y}{h}, \quad \xi_0 = \frac{y_0}{h}, \quad w = \frac{u}{U}, \quad w_0 = \frac{u_0}{U}, \quad \bar{t} = \frac{\mu_p t}{\rho h^2}$$

Eqs. (1)-(6) take the form

$$\frac{\partial w}{\partial \bar{t}} = \frac{\partial^2 w}{\partial \xi^2} + S, \tag{7}$$

$$\frac{dw_0}{d\bar{t}} = -\frac{S\xi_0}{1-\xi_0}, \tag{8}$$

$$w[\xi_0(\bar{t}), \bar{t}] = w_0(\bar{t}), \quad \frac{\partial}{\partial \xi} w[\xi_0(\bar{t}), \bar{t}] = 0,$$

$$w(0, \bar{t}) = \frac{P \frac{\partial}{\partial \xi} w(0, \bar{t})}{1 + P \frac{\partial}{\partial \xi} w(0, \bar{t})}, \tag{9}$$

$$w(\xi, 0) = 1, \quad w_0(0) = 1, \quad \xi_0(0) = 0. \tag{10}$$

Here $S = \tau_0 h/\mu_p U$ is the Saint Venant parameter; $P = k/h$.

Because of the mobility of the boundary region $\xi_0(\bar{t})$, there are serious mathematical difficulties in obtaining an accurate solution of Eqs. (7)-(10).

The velocity distribution approximates to the form

$$w(\xi, \bar{t}) = \begin{cases} A(\bar{t}) + B(\bar{t})\dfrac{\xi}{\xi_0(\bar{t})} + C(\bar{t})\dfrac{\xi^2}{\xi_0^2(\bar{t})}, & 0 \leqslant \xi \leqslant \xi_0(\bar{t}), \\ w_0(\bar{t}), & \xi_0(\bar{t}) \leqslant \xi \leqslant 1. \end{cases} \tag{11}$$

The coefficients A, B, and C are determined from the boundary condition (9):

$$C = \frac{\xi_0}{4P} - \frac{1-w_0}{2} - \sqrt{\frac{\xi_0^2}{16P^2} + \frac{(1-w_0)^2}{4} + \frac{\xi_0(1+w_0)}{4P}},$$
$$A = C + w_0, \quad B = -2C. \tag{12}$$

To find the relation between the required functions $\xi_0(\bar{t})$ and $w_0(\bar{t})$, Eq. (7) is satisfied in the mean:

$$\int_0^{\xi_0(\bar{t})} \frac{\partial w}{\partial \bar{t}} d\xi = \int_0^{\xi_0(\bar{t})} \frac{\partial^2 w}{\partial \xi^2} d\xi + S\xi_0(\bar{t}). \tag{13}$$

Taking into account Eqs. (8), (11), and (12), Eq. (13) is integrated to give

$$\frac{d\xi_0}{d\bar{t}} = \frac{SC_1\xi_0^3 + 3S\xi_0^2 - 6C\xi_0 + 6C}{\xi_0(1-\xi_0)(C_2\xi_0 + C)}, \tag{14}$$

where

$$C_1 = \frac{\partial C}{\partial w_0} = -\frac{1}{2} - \frac{1}{4}\frac{w_0 - 1 + \xi_0/2P}{\sqrt{\frac{\xi_0^2}{16P^2} + \frac{(1-w_0)^2}{4} + \frac{(1+w_0)\xi_0}{4P}}},$$

$$C_2 = \frac{\partial C}{\partial \xi_0} = \frac{1}{4P} - \frac{1}{8P}\frac{w_0 + 1 + \xi_0/2P}{\sqrt{\frac{\xi_0^2}{16P^2} + \frac{(1-w_0)^2}{4} + \frac{(1+w_0)\xi_0}{4P}}}. \tag{15}$$

From the system of two ordinary differential equations (8) and (14) and the initial condition (10), it is possible to determine $w_0(\bar{t})$ and $\xi_0(\bar{t})$ and hence to obtain an approximate solution of the problem. Numerical integration of Eqs. (8), (14), and (10) is used.

Some results of the calculation are shown in Figs. 1-3. In the initial motion of the film, the viscoplastic region is broadened; it reaches a maximum extent ξ_0^{max} and then decreases (Figs. 1b and 2b). The presence of slip at the wall reduces ξ_0^{max} and increases the rate of motion of the quasisolid region (Fig. 1a) and the time of film flow $\bar{t}_1$ (Fig. 3). At the instant $\bar{t}_1$ the viscoplastic region disappears and motion of the film ceases.

For small values of $\bar{t}$, the basic characteristics of the motion are

$$\xi_0 = \sqrt{8\bar{t}} + Q(\sqrt{\bar{t}}), \quad w = 1 - \frac{4\sqrt{2}}{3}S\bar{t}^{\frac{3}{2}} - Q(\bar{t}_1^{\frac{3}{2}}).$$

The mass of liquid shaken loose per unit width of the film is given by

$$Q = \int_0^{\bar{t}_1}\int_0^1 w(\xi, \bar{t})\,d\xi d\bar{t} = \int_0^{\bar{t}_1}(w_0 + \frac{1}{3}C\xi_0)\,dt.$$

It follows from Fig. 3 that the presence of slip at the wall increases Q and increase in S leads to decrease in the mass of liquid shaken loose.

LITERATURE CITED

1. V. I. Baikov, Izv. Akad. Nauk BelorusSSR, Ser. Fiz.-Énerg. Nauk, No. 3 (1974).